Risk and Liquidity Requirements in a Model of Fractional Reserve Banking*

Stephan Imhof
Swiss National Bank

Cyril Monnet
University of Bern
Study Center Gerzensee
Swiss National Bank

Shengxing Zhang
London School of Economics

November 2017

Abstract

We study the implications of deposit making on risk taking. We capture several important features of bank lending: Banks finance firms by creating deposits. Firms use these deposits as means of payments. Deposits carry a risk premium as long as they are not insured or only partially backed by liquid assets. Optimally the amount of deposits is restricted by some reserves or liquidity requirements. The optimal combination of reserve or liquidity requirements and inflation trades-off investment and risk. Increasing these requirements or inflation makes loans to the private sector more expensive. This induces borrowers to take more risk. But they also invest less when loans are more expensive. As a result leverage declines. This induces borrowers to take safer decision.

*For comments and suggestions, we thank seminar and conference participants at the 2017 Money and Liquidity conference at the bank of Canada, 2017 Swiss National Bank research conference, the Reserve Bank of New Zealand, and the University of Zurich, and especially Juerg Blum, Jonathan Chiu, Michael Kumhof (discussant), Steven Ongena, Jean-Charles Rochet, Stephen Williamson (discussant), Randall Wright, and Cathy Zhang. The views are the one of the authors and do not necessarily reflect the views of the Swiss National Bank.

1 Introduction

A fundamental question in monetary economics and banking is whether an economy that allows banks to freely create deposits is more or less stable than the same economy relying solely on deposits fully backed by reserves.[1] The famous Chicago plan called for 100% reserve requirements at a time when the Great Depression gave ammunitions to those arguing for limiting the use of inside money. The Great Recession revived the academic and policy debate: Movements to reform the monetary system towards 100% reserve requirements exist in more than twenty countries.[2] Switzerland will vote in 2018 on a binding national referendum initiated to drastically limit banks' ability to create unbacked deposits (a.k.a. the VollGeld initiative).

The theoretical and empirical literature is large and below we only review its most recent development, but let us mention two recurrent themes. A system relying on the free creation of deposits is arguably more efficient because banks are more flexible to respond to loan demand (e.g. Williamson, 1999). But this system is inherently unstable because it allows multiple equilibria, which opens the door to exotic dynamics, cycles, and crashes (e.g. Sanches, 2015). One puzzling aspect of the literature is that risk is missing from the analysis: Banks and their borrowers do not engage in risk taking activities and in equilibrium banks or firms do not take any risk. Rather and as in the seminal paper by Diamond and Dybvig (1981), it is the source of fundings that is fragile. In this paper, we analyze the stability properties of an economy with deposit creation by putting the risk-taking decision of borrowers at the center of our analysis.

More precisely, we introduce the moral hazard of risk taking in an otherwise standard monetary model with banks. We model risk as a continuous choice variable. As a result, there is an optimal amount of risk that borrowers should take. So, even at the optimum, projects will fail and so may banks that financed those projects. However, moral hazard and limited liability implies that borrowers take too much risk in equilibrium and the more indebted

[1] We will use deposits and deposits to mean "inside money". We will use cash, currency, reserves or money to mean "money".

[2] See http://internationalmoneyreform.org.

they are the more risk they take. When borrowers face a low loan rate, they will tend to borrow more, increasing their indebtedness, thus taking more risk. In this context, we study if and how a fractional reserve banking system can help achieve the (constrained) optimal level of debt and risk taking.

We show that liquidity requirements combined with inflation exploit a trade-off between risk-taking and the level of investment. In an inflationary environment, liquidity requirements are costly for banks and they recoup this cost by adjusting the loan rate they charge borrowers. So borrowers will tend to reduce the amount they borrow and their leverage. Since they have more at stake, they increase the quality of their project and take less risk. In general, we show that when the real loan rate is low, the firm takes on a higher leverage and chooses riskier investment. This trade-off is reminiscent of the two themes we mentioned earlier.

However, another effect of imposing higher reserve requirements when there is no deposit insurance is to increase the return on deposits when borrowers fail. As a result, deposits backed by more reserves can buy more: the borrower faces a better effective loan rate. Again, this can improve stability by lowering the terms of loans but this may also hamper it by inducing more borrowing.

The optimal combination of reserve requirement and inflation trades-off these effects. The Friedman rule or zero reserve requirement is not necessarily optimal, as it would induce too much leverage. In spite of being the safest system, fully backed deposits may not be optimal as it can reduce leverage too much.

The rest of the paper is organized as follows. We present the model in Section 2 and we derive the equilibrium in Section 3. Section 4 contains several extensions such as the effect of bail-out policies, and the consequences of capital requirements. We summarize the recent literature in Section 5. The last section summarizes the findings and concludes.

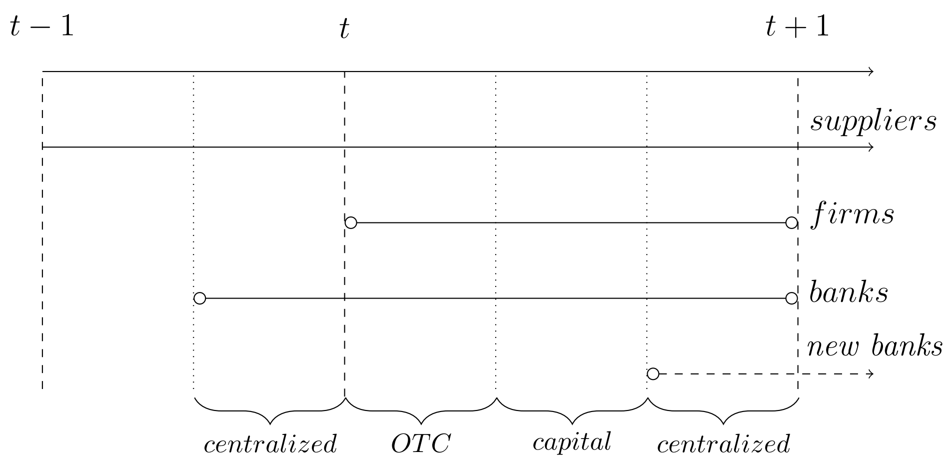


Figure 1: Births and deaths.

2 The Model

The model is a version of Rocheteau, Wright, and Zhang (2016). Time $t = 1, 2, ...$ is discrete and continues forever. Each period is divided in two subperiods. There are two goods, capital and the numeraire, that are not storable. There are three types of risk neutral agents, each with measure one, firms, suppliers, and bankers. Suppliers are infinitely lived, but firms and banks live finite life. In each period, a measure one of firms is born at the beginning of the first subperiod and die at the end of the second subperiod. Also, a measure one of bankers is born in the second subperiod of each period and die in the second subperiod of the next period. Therefore, we have an overlapping generation structure of bankers. As will be clear, this implies that firms cannot build equity so that they have to borrow, and bankers will be short-sighted but will be able to build equity when young. Figure 1 shows the life span of each agents in the economy.

Preferences and technologies Suppliers are endowed with two linear technologies: In the first subperiod, they can transform hours worked one-for-one into capital,[3] while in the second subperiod they can transform hours worked one-for-one into the numeraire. Young bankers are also endowed with two technologies: they can transform hours worked one-for-

[3]Conveniently, this implies that we can also interpret capital as labour.

one into the numeraire, they can also create deposits (banknotes) that cannot counterfeited and can be traded.[4] A deposit is a promise to 1 unit of the numeraire in the next subperiod. Old bankers cannot work, but they can commit to repay their debt and they are endowed with a debt enforcement technology.

A fraction $\lambda \in [0, 1]$ of firms are endowed with the following technology

$$f(k, q) = \begin{cases} F(k) & \text{with probability } q, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

where $F(k)$ is a neoclassical production function that transforms capital k into some numeraire goods. We assume $F(k)$ is homogeneous of degree σ. The firm chooses $q \in [0, 1]$ by suffering a cost $q^2F(k)/2$. So a higher q makes the positive outcome more likely but also lowers the surplus from production. We think of q as the quality of the project but also as inversely related to the level of risk taking.

All in all, suppliers produce the capital good k in the first subperiod. It is then bought by firms and invested with a return in the second subperiod. The numeraire is only produced and/or consumed in the second subperiod.

Preferences of suppliers and bankers are represented by the utility function $U(c, h) = c - h$ where $c \geq 0$ is the consumption of the numeraire and $h \geq 0$ is hours worked. Firms' preferences for consumption of the numeraire is just $u(c) = c$. Banks and suppliers discount the future at a rate $\beta \in (0, 1)$.

Markets In the first subperiod, there is an OTC market for banking services with search and bargaining, followed by an interbank market and a Walrasian market for capital. As not all firms are productive, some banks are unmatched and will become lenders on the interbank market. In the second subperiod, there is a frictionless centralized market where agents produce and/or consume the numeraire and settle debts.

[4] Young bankers can create equity by producing and selling the numeraire for money. In Section 4.4 we show how to relax the assumption that young bankers can produce the numeraire, while still being able to raise equity.

Firms cannot commit to repay suppliers and they have no equity. So firms have to borrow assets from a bank and use them to purchase capital. There are two types of assets. We already mentioned deposits. There is also money (currency, or reserves) which stock evolves according to $M_{t+1} = (1+\pi)M_t$. The price of money in terms of the numeraire is v_t. In stationary equilibrium $v_{t+1}M_{t+1} = v_t M_t$, so $v_t = (1+\pi)v_{t+1}$. We let the nominal rate be $i = (1+\pi)/\beta - 1$, and r be the interest rate banks earn on required and excess reserves. We assume $i > r$ which implies that banks want to economize on their reserve holdings. Therefore, absent any reserve requirements, banks will not want to hold any money, as we show in the appendix.

Timing The timing is as follows. In each period, the OTC market for banking services open first. There, firm-bank pairs are formed. One firm is matched randomly with one bank with probability α where $\alpha \leq 1$ is the measure of operating banks in the economy.[5] As a result, a measure $\alpha\lambda$ of firms are productive and can possibly obtain a loan from a bank. With probability $(1-\alpha)\lambda$ a firm is productive but does not meet a bank. Once matched, the bank and the productive firm bargain over the terms of the loan in a way we describe below. Concurrently banks with too little reserves can borrow from banks with too much reserves in the interbank market. Then, firms who managed to obtain a bank loan use it to purchase capital from suppliers in the Walrasian market for capital. Then they invest capital and choose the quality of their project.

In the second subperiod, successful firms repay their bank loans. While it would be natural (and feasible) to assume that firms sell their output for deposits/cash and then repay their loans, we keep things as simple as possible and we assume that firms repay their (now old) banker by transferring some of their output.[6] The bank redeems its deposits using reserves and some of the output from the firm. Unsuccessful firms cannot pay back their banker who then only has reserves to redeem their deposits. The banker fails when reserves are not enough to pay the par-value of the deposits. In this case, the holders of deposits are paid pro-rata. Because old bankers cannot produce the numeraire goods, deposits can be risky if

[5] We take α as given for most of the paper, and we endogenize it in one of our extensions.

[6] See Rocheteau, Wright, and Zhang (2017) for details.

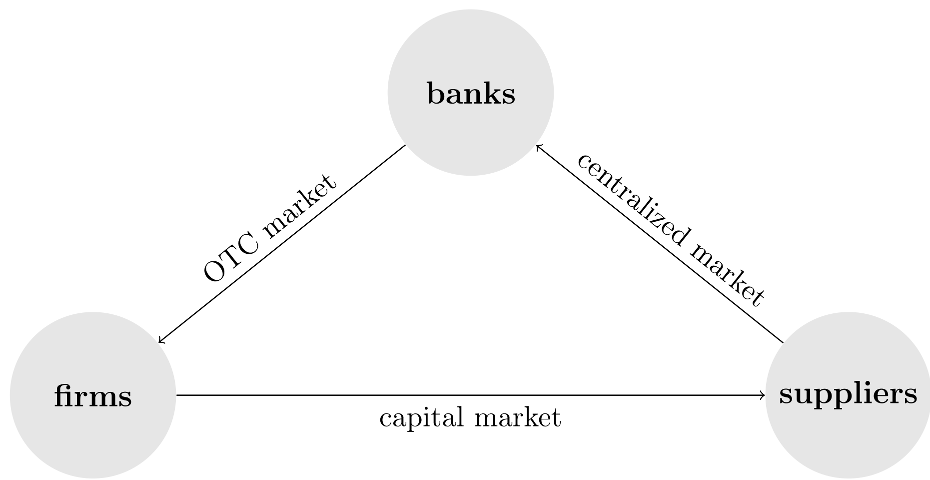


Figure 2: deposits circulation

the banker does not hold enough reserves. We assume the failed bank is replaced by a new bank. One of our extensions modifies this assumption so that there is a real cost of failure.

Finally, suppliers and successful banks may hold reserves or currency but have little use for it. They can sell them to a young banker. The young banker has the ability to produce the numeraire and so can build equity in the form of reserves (this is sweat equity) for the upcoming loan market.

Figure 2 shows the circulation of deposits in all three markets. We now describe each market in more details and in chronological order.

2.1 Markets for bank loans and reserves

A productive firm needs a bank loan because he has no equity and he is not trustworthy to obtain credit directly from suppliers. A bank grants a loan using deposits (since $i > 0$ banks only holds money to satisfy their reserve requirements, if any). Let p denote the *real* price of deposits in the market for capital.[7] Then firms need pk of deposits for them to buy k units of capital. The bank charges ϕ for this loan. We will assume that the quality of the

[7]These prices may differ because we do not assume deposit insurance (however, see an extension). So banks may default on their deposits but the CB does not default on money.

project is contractible.[8] Hence a bank loan is a list (pk, ϕ, q). With this loan, a firm buys k units of capital from suppliers and pay them the real amount pk with deposits. To simplify notation, we will say that a bank loan is a list (k, ϕ, q).

Reserve/liquidity requirements Banks may face liquidity requirements in the form of a reserve requirement. The regulator pays an interest $r \geq 0$ on required and excess reserves.[9] We assume that a young bank who issued deposits worth k has to set aside enough reserves (in the form of money) to be able to pay at least $\tilde{\tau}k$ when old to its depositors, where $\tilde{\tau} \in [0, 1]$ is a policy variable. So reserve requirements differ from capital requirements insofar as reserve requirements are not invested and can be used to pay holders of deposits if investments turn bad. So banks who extend a contract (k, ϕ) must hold reserves R so as to satisfy the constraint (in real terms)

$$\tilde{\tau}k \leq (1 + r)R. \tag{2}$$

Again, we assume $i > r$ so that the reserve constraint (2) binds. Also, for simplicity we use the normalization $\tau \equiv \tilde{\tau}/(1 + r)$.

Interbank market Banks can change their holdings of reserves in the interbank market. This market is organized as a Walrasian market and i_m is the market clearing rate. Banks can increase their current holdings m^b by borrowing b reserves on the interbank market, or they can lend ℓ reserves when they have excess reserves. For simplicity we assume that interbank loans are unsecured and so junior to any other loans in case of bankruptcy. Hence, if the firm fails, its suppliers get paid first by the bank from whatever amount of reserves it has.

Since we consider fractional reserves ($R \leq pk$), a failing bank with a binding reserve requirements can only (partially) reimburse holders of deposits (suppliers) and not its junior interbank loans. In this case, banks that are lending on the interbank market expect their

[8] We worked out a version of this model where the quality of the project is not contractible. The results, available from the authors, are qualitatively identical.

[9] The case where $r < 0$ is available from the authors.

interbank loans to fail with probability Q in which case they get nothing. Therefore the outside option of a bank holding reserves m is just $Q(1+i_m)m$.

Banks' participation constraint We can now define the bank's participation constraint given contract (k, ϕ), as

$$q\left[pk + \phi + (1+r)\tau k - pk - (1+i_m)(\tau k - m)^+ + Q(1+i_m)(m - \tau k)^+\right] \quad (3)$$

$$+(1-q)\left[Q(1+i_m)(m-\tau k)^+ + (1+r)\tau k - pk\right]^+ \geq Q(1+i_m)m \quad (4)$$

where $(x)^+$ equals x whenever $x \geq 0$ and zero otherwise. If the firm fails, the bank's only resource from which it has to pay its deposits is the net income from lending its excess reserves (if any) in the interbank market. The bank goes bankrupt and naturally gets zero payoff iff it does not have enough reserves to pay its liabilities. If the firm succeeds, the bank gets paid the principal and its ϕ from which it redeems its deposits and pay its interbank loans, if any. Again, when $i > r$, the bank never holds *excess* reserves, so that $m < \tau k$, but borrows the shortfall in the interbank market. Then we can simplify the participation constraint as

$$q\left[\phi + (1+r)\tau k - (1+i_m)(\tau k - m)\right] \geq Q(1+i_m)m \quad (5)$$

Loan contract The choice of q is contractible by the banker (but not by suppliers) so that there is no moral hazard between the firm and the banker in this version of our model. In addition, we assume the banker has no bargaining power, so a loan contract is a tuple (k, ϕ, q) that maximizes the firm's payoff,

$$\max_{k,\phi,q} q[F(k) - \phi - pk] - \frac{1}{2}q^2 F(k),$$

subject to the bank's participation constraint

$$q\left[\phi + (1+r)\tau k - (1+i_m)(\tau k - m)\right] \geq Q(1+i_m)m.$$

Our assumption that loan contracts give the entire surplus to firms implies that the moral hazard problem originating from lending is minimized. Any other surplus sharing rule would make the moral hazard problem even more acute and would reinforce our result. Also, introducing moral hazard between the firm and the bank would only reinforce our result.

Once banks loans are agreed upon and required reserves are set aside, the market for capital opens.

2.2 Capital market

The demand for capital is given by the bank loan contract. To determine the supply of capital, we turn to the problem of suppliers in the capital market. Obviously, suppliers are aware of the moral hazard problem and they expect each firm (and their bank) to fail with probability $1 - Q$. In addition, when the bank fails, they expect to get the reserves held by the bank. The capital market being Walrasian, suppliers are able to perfectly diversify by selling capital to every productive firms.[10] Hence, the problem of a supplier entering the capital market with m^s real units of money is

$$V^s(m^s) = \max_{k \geq 0} \{m^s - k + (1 - Q)(1 + r)\tau k + Qpk + W^s(0)\} \tag{6}$$

where k is the capital sold for deposits at (real) price p, and we already anticipated that the value of net worth in the second subperiod $W^s(\omega)$ is linear. The first order condition gives

$$p = \frac{1 - (1 - Q)(1 + r)\tau}{Q} \tag{7}$$

Clearly, deposits carry a risk premium unless the banks hold 100% reserves, that is $\tilde{\tau} = 1$.

[10] The ability to diversify plays no role in this model where suppliers are risk neutral.

2.3 Centralized market (CM)

In the last centralized market, successful firms settle their debt and their bank redeems their deposits. They consume whatever is left. Suppliers consume their real net worth ω and solve the following savings problem

$$W^s(\omega) = \omega + T + \max_{m^s \geq 0}\{-(1+\pi)m^s + \beta V^s(m^s)\}, \tag{8}$$

where T is a real transfer (possibly negative) and m^s is the real amount of money the suppliers choose to carry over to the next period. So using the expression for $V^s(m^s)$ in (6) which is linear in m^s, the problem of suppliers becomes

$$\max_{m^s}\{(-(1+\pi)+\beta)m^s\}$$

so that $m^s = 0$ iff $(1+\pi)/\beta = 1+i > 1$, it is indeterminate when $i = 0$ and it is infinite whenever $i < 0$. So the only equilibrium is one where the nominal interest rate is positive, $i \geq 0$.[11] Given this result, we will naturally assume that suppliers hold no real balances since they have no liquidity needs.

Let us now turn to the problem of young banks in the CM. Banks will choose real balances m^b to maximize their lifetime net worth, given they obtain no surplus when they are lending to a firm:

$$V^b(m) \quad = \quad \max_m -(1+\pi)m + \beta Q(1+i_m)m$$

The reader should notice the existence of a hold-up problem when there is reserve requirement and $i > r$: While banks incur the cost of bringing real balances in the loan market, they do not obtain any surplus from it. Therefore, if the interbank market was absent, there would not be any equilibrium with lending because banks would never incur the cost of acquiring real balances knowing firms would anyway grab the surplus this generates. However, the

[11] Equilibrium with $i < 0$ are possible whenever the regulator remunerates reserves of banks and suppliers at $r < 0$.

interbank market gives banks a viable (outside) option with unit payoff $Q(1+i_m)$. The firm has to promise the bank at least this, and it is sufficient for an equilibrium with lending to exist. Still, in equilibrium, it must that

$$1 + i_m = \frac{1+\pi}{Q\beta} = \frac{1+i}{Q} \tag{9}$$

and the bank is indifferent as to the amount of real balances it brings (it would obtain the same payoff by bringing none).

2.4 Equilibrium

We can now define a symmetric steady state equilibrium.

Definition 1. A symmetric steady state equilibrium is a list consisting of loan contracts (k, ϕ, q), project quality Q, prices p, v, i_m, r, choice of real balances m, inflation π, and reserve requirement $\tau = (1+r)\tilde{\tau}$, such that: given $\tilde{\tau}$, prices π,v, r, i_m, and p, the amount of capital k, ϕ, and q solves the bargaining problem, m is given by market clearing, p is given by (7), i^m is given by (9), the market for balances clear $m = vM$, and aggregate quality is consistent with individual choices $Q = q$.

Since we consider a symmetric equilibrium, the interbank market clearing condition is

$$\underbrace{(1-\lambda)m}_{\substack{\text{reserves from} \\ \text{non-lending banks}}} = \lambda \underbrace{(\tau k - m)}_{\substack{\text{reserve deficit} \\ \text{of a lending bank}}}$$

so that

$$m = \lambda \tau k. \tag{10}$$

3 Equilibrium characterization

In this section, we characterize the equilibrium when $i > r$ so that the regulator remunerates reserves but not enough to compensate banks for the cost of holding them. To do this, we first solve the bargaining problem determining the equilibrium loan contract. Since the participation constraint of the bank binds, we can eliminate ϕ from the firm's problem to obtain the following problem

$$\max_{k,q} q\left[F(k) - pk + (1+r)\tau k - (1+i_m)(\tau k - m)\right] - Q(1+i_m)m - \frac{1}{2}q^2 F(k).$$

and the first order conditions for k and q are, respectively,

$$k: \qquad F'(k)\left(1 - \frac{1}{2}q\right) = p + (i_m - r)\tau \tag{11}$$

$$q: \qquad (1-q)\,F(k) = pk + (i_m - r)\tau k - (1+i_m)m \tag{12}$$

The funding cost of lending the amount of deposit necessary to purchase an additional unit of capital consists of the cost of redeeming these additional deposits p, as well as the marginal opportunity cost of the required reserves τ: they earn r but could be lent for i_m The marginal cost of higher quality consists of the marginal effort cost, i.e. $-qF(k)$, as well as the fact that the payment to the bank has to be made more often. (12) clearly shows that q is decreasing with the bank/firm's level of indebtedness as measured by $pk + (1+i_m)(\tau k - m)$ but increasing with the interest rate on reserves.

Replacing the equilibrium value for the interbank rate, the price of deposits, as well as m, and arranging we obtain how the (optimal) contract responds to changes in policy variables, i and τ but also in the market perception of risk Q.

$$F'(k)\left(1 - \frac{1}{2}q\right) = \frac{1 + (i-r)\tau}{Q} \tag{13}$$

$$(1-q)\,\frac{F(k)}{k} = \frac{1 + (i-r)\tau - (1+i)\lambda\tau}{Q} \tag{14}$$

(14) implies that $q < 1$ iff $1 + (i-r)\tau > (1+i)\lambda\tau$ which we will assume from now onward.

Holding q constant, (13) says that investment will increase with market perception Q, but will decrease with inflation and reserve requirements. Similarly, holding k constant, (14) says that quality is increasing with Q, and decreasing with inflation. We have the following result,

Lemma 1. *Equilibrium investment k is always increasing with market perception Q while the equilibrium choice of quality q is not a function of Q.*

Proof. To see this, replace (13) in (14). Making use of $F'(k) = \sigma F(k)/k$ and arranging, we obtain an expression for k as a function of Q,

$$F'(k) = (2-\sigma)\left[\frac{1+(i-r)\tau}{Q}\right] + \sigma\frac{(1+i)}{Q}\lambda\tau$$

and clearly, k is always increasing with Q. Using this expression back in (14), we find

$$1 - q = \frac{\sigma\left[1+(i-r)\tau - (1+i)\lambda\tau\right]}{(2-\sigma)\left[1+(i-r)\tau\right] + \sigma(1+i)\lambda\tau} \tag{15}$$

As stated in the lemma, quality is not a function of market perception. □

This result may be surprising because (14) implies that quality is increasing with Q given k. Obviously, the reason is that the market perception of quality reduces the risk premium on deposits and interbank loans, which makes investing cheaper. As a result, investment increases but just enough in order to maintain the indebtedness level of the firm toward the bank. Therefore the firm takes the same level of risk. This is a key mechanism of our paper: By reducing the risk premium, market perception makes funding cheaper, which induces firms to invest more. In equilibrium $q = Q$, and (15) defines the equilibrium level of quality.

Proposition 1. *When $1+(i-r)\tau > (1+i)\lambda\tau$, a unique equilibrium exists where the reserve requirement always binds and the interbank market is active. The equilibrium quality Q is*

$$Q = 1 - \frac{\sigma\left[1+(i-r)\tau - (1+i)\lambda\tau\right]}{(2-\sigma)\left[1+(i-r)\tau\right] + \sigma(1+i)\lambda\tau} \tag{16}$$

and investment k solves

$$F'(k) = \frac{[(2-\sigma)(1+(i-r)\tau) + \sigma\lambda(1+i)\tau]^2}{2(1-\sigma)[1+(i-r)\tau] + 2\sigma\lambda(1+i)\tau} \tag{17}$$

Proof. The proof merely consists of re-arranging (13) and (14), setting $q = Q$. The assumption guarantees that $Q \in [0,1]$ and requires that r (or i) cannot be too high. $\square$

To illustrate how the equilibrium with deposits changes with policy variables, we replace the equilibrium condition $q = Q$ in equations (13) and (14) to obtain the equilibrium investment and risk curves,

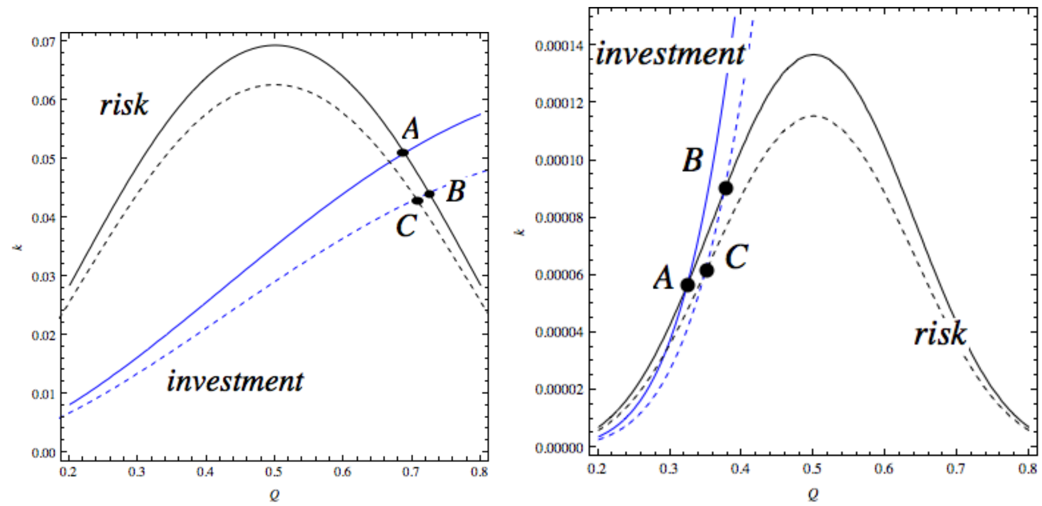
$$\left(1 - \frac{1}{2}Q\right) F'(k) = \frac{1+(i-r)\tau}{Q} \qquad (investment) \tag{18}$$

$$(1-Q)\frac{F(k)}{k} = \frac{1+(i-r)\tau}{Q} - \frac{(1+i)}{Q}\lambda\tau \qquad (risk) \tag{19}$$

Figure (3a) shows both curves. The equilibrium with no inflation is at point A. As inflation rises from r say, the marginal cost of capital increases, everything else constant. Hence given Q, investment will decline, and the investment-curve shifts down. This induces a decrease in investment, which, if nothing would change would induce a move down on the risk-curve from A to B: As leverage declines, quality increases and risk drops. However, inflation induces the bank to charge a higher interest rate to the firm (the risk-curve shift down), so that the firm chooses a lower quality. So the equilibrium moves from B to C. In the new equilibrium, there is lower investment. The reduction in investment is sufficient to undo the increase in cost of funds, so that leverage still declines. Our result implies that the investment effect due to the change in inflation is always stronger than the direct effect on quality; as a result Q always increases with inflation. Intuitively, this is due to the presence of a positive feedback loop: Because quality increases, deposits are now safer, so the risk premium in p declines. This contributes to a further reduction in leverage and higher average quality.

In addition, there are several remarks worth making on Proposition 1.

- When $i > r \geq 0$ and $\tau > 0$ then reserve requirements and inflation are substitutes in affecting risk. However, it must be that $\tau > 0$ for inflation to impact risk. Indeed,



(a) Equilibrium Q, k. (b) Equilibrium with high σ.

the bank only lends deposits to the firm and keeps just enough reserves to satisfy its requirements. Setting $\tau = 0$, it is obvious that Q is independent of inflation or the interest rate on reserves. Hence, for inflation to affect risk-taking it must be that $\tau > 0$.

- When $\tau > 0$, the comparative statics of Q with respect to its arguments are

$$\frac{\partial Q}{\partial i} > 0, \ \frac{\partial Q}{\partial \tau} > 0, \text{ and } \frac{\partial Q}{\partial r} < 0.$$

  So more inflation and higher reserve requirements reduce risk-taking, but higher interest rate on reserves increases risk taking. The intuition for the last result is simple: When the interest rate on reserves is higher, the bank has more to lose by lending to the firm (e.g. in case the firm fails, the bank loses the interest on the reserves it holds) and so requires a higher payment. This reduces the firm's incentives to exert an effort and Q drops.

- The reaction of investment k to the policy variable depends on σ. While Figure 3a shows that k drops with inflation, the decline in investment is not ineluctable. In particular, the investment curve is directly susceptible to σ. As illustrated in Figure 3b, increasing σ shifts the investment curve down, but also makes it steeper, so that the investment-curve will cross the risk-curve when it is increasing. Then, as we show below, increasing inflation might increase equilibrium investment.

- From (14), we can write $q = 1 - (\text{effective rate} \times k)/F(k)$, where $effective\ rate = \frac{1+(i-r)\tau}{Q} - \frac{(1+i)}{Q}\lambda\tau$. Therefore, the sensitive of quality with respect to investment is related to σ since

$$\frac{\partial q}{\partial k} = -\frac{(\text{effective rate})}{F(k)}(1-\sigma).$$

  Hence, as σ increases towards 1, the choice of quality becomes less (negatively) sensitive to investment. In the limit, q is totally insensitive to the investment level.

- We can show that if σ is high enough, then investment k increases in i and τ whenever $(1+i)\tau$ is sufficiently small. In words, if money is relatively cheap for banks to hold $(i = r = 0)$, then investment increases with reserve requirement. This is intuitive: Since there is little inflation, the cost to raise reserve requirement is small for the bank. But higher reserves, implies that deposits are safer. As a result, firms can now invest more for a given loan size. As σ is large, quality does not respond much to investment and so does not (fully) undo the beneficial effects of higher reserves. Similarly, investment increases when there is little reserve requirement and inflation is raised from the Friedman rule. This seems counterintuitive at first because we concluded above that Q increases with inflation thanks to lower leverage. But a general equilibrium effect plays through the decline in the risk premium on inside money. As it is cheaper to fund firms, and as τ is small, banks do not suffer much from the rising inflation and the overall cost of funds for the firm can decline. Our result shows that this decline in the cost of funds, while enticing the firm to invest more, can be large enough to reduce leverage.

3.1 Welfare

In this section we study the welfare consequences of the risk-investment trade-off. As all agents are risk neutral, welfare is given by aggregate output net of the cost of producing the investment good and the firm's cost of effort,

$$\mathcal{W} = \alpha\lambda \left[Q\left(1 - \frac{1}{2}Q\right) F(k) - k \right].$$

A planner seeking to maximize welfare will choose investment k^* and quality Q^* to maximize $\mathcal{W}$. The first order conditions are

$$Q^*\left(1-\frac{1}{2}Q^*\right)F'(k^*)=1 \tag{20}$$

and

$$Q^*=1$$

So that $F'(k^*)=2$.

We now determine an expression for welfare in equilibrium. First notice that there is room for policy actions. Indeed, comparing (20) and (18), it would be optimal to set $i=\tau=0$ if this was not affecting risk taking. However there is too much risk taking when $\tau=0$ since then

$$Q=1-\frac{\sigma}{2-\sigma}<1.$$

Also, $Q<1$ and (21) implies that the market equilibrium is characterized by underinvestment. To obtain an expression for welfare in equilibrium, notice we can rewrite the investment curve (18) as,

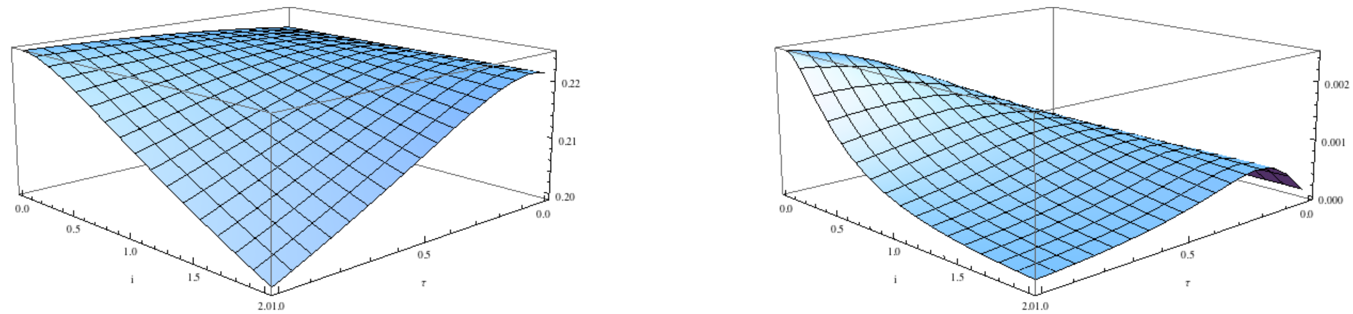
$$Q\left(1-\frac{1}{2}Q\right)F(k)-k=k\left[\frac{1}{\sigma}-1+\frac{1}{\sigma}(i-r)\tau\right] \tag{21}$$

which gives a simple expression for welfare. Clearly if σ is high enough that k is increasing with i or τ, then increasing i is optimal. The optimal choice of i and τ maximizes the RHS of (21).[12] Figure 3 shows welfare for different level of inflation and reserve requirements, for different values of σ. The figure shows that independently of σ the optimal level of reserve requirement is $\tau=1$ while the optimal inflation rate is $i=r$. As inflation rises however, the optimal reserve requirement ratio falls. We show this result in the following Proposition.

[12]The first order conditions give

$$\begin{aligned}\left(\frac{1}{\sigma}-1+\frac{1}{\sigma}(i-r)\tau\right)\frac{dk}{di}+k\frac{\tau}{\sigma} &= 0\\ \left(\frac{1}{\sigma}-1+\frac{1}{\sigma}(i-r)\tau\right)\frac{dk}{d\tau}+k\frac{(i-r)}{\sigma} &= 0\end{aligned}$$

which can be solved for i and τ.



(a) $\sigma = 0.2$ (b) $\sigma = 0.8$

Figure 3: Welfare.

Proposition 2. *Suppose* $i \simeq r$ *then* $\tau = 1$ *is optimal. Suppose* $i - r > 0$ *and large, then* $\tau < 1$ *is optimal.*

Proposition 2 shows the trade-off between quality and output: When inflation is high, investment is already low and increasing reserve requirements, while improving quality, would also make investment more costly. Thus it is optimal to reduce reserve requirements. Alternatively, when inflation is low, holding reserves is not so costly and investment is relatively high, so that increasing reserve requirement is optimal, as quality concerns dominate.

4 Extensions

4.1 Operating banks α

Most, if not all, regulators will agree that it is socially costly for any banks to fail. One such cost is the disruption of the payment system, and some argue that there is a loss of expertise. In this section, we get to the idea that bank failure is costly by assuming that if a bank fails, it is replaced by a new bank, but with a one period lag (the time necessary to unwind the bank). As a result, bankers will not fully internalize the cost of their default. Our qualitative results that reserve requirements can be welfare improving do not depend on this assumption, although it may affect the quantitative predictions of the model. In doing so, we also endogenize α in this section.

We now compute the number of operating banks in period t. Since banks that financed a failing firm lose their license for one period, the number of operating banks in period t is α_t:

$$\alpha_t = Q \underbrace{\lambda\alpha_{t-1}}_{\text{\#b with a match}} + \underbrace{(1-\lambda)\alpha_{t-1}}_{\text{\#b with no match}} + \underbrace{(1-\alpha_{t-1})}_{\text{\#b not operating last period}}, \text{ or}$$

$$1 - \alpha_t = (1 - Q_{t-1})\lambda\alpha_{t-1}$$

So in steady state $\alpha \equiv \alpha_t = \alpha_{t-1}$,

$$\alpha = \frac{1}{1 + \lambda(1 - Q)}. \tag{22}$$

If operating banks always meet productive firms (i.e. $\lambda = 1$) then $\alpha = 1/(2 - Q)$. Naturally, if $Q = 1$ then all banks are operating, while less banks are operating otherwise. Then all of our equilibrium analysis goes through. Welfare is given by aggregate output net of the cost of producing the investment good and the firm's cost of effort,

$$\mathcal{W} = \alpha\lambda\left[Q(1 - \frac{1}{2}Q)F(k) - k\right]$$

where α is now given by (22). So optimal investment k^* and quality Q^* solve

$$Q^*\left(1 - \frac{1}{2}Q^*\right)F'(k^*) = 1 \tag{23}$$

and $Q^* = 1$.[13] The unit upper bound on quality binds because the planner would require a higher quality because he internalizes the cost of bank failure.

[13] The first order condition is

$$\frac{\lambda}{[1 + \lambda(1 - Q)]^2}\left[Q(1 - \frac{1}{2}Q)F(k) - k\right] + \frac{1}{1 + \lambda(1 - Q)}(1 - Q)F(k) = 0$$

which simplifies in the expression above.

4.2 (Anticipated) bail-outs

In this section, we consider the case where the government decides to bail-out failing banks when it does not remunerate reserves, $r = 0$. This means that the government will pay off all of the failing bank's liabilities by taxing suppliers in a lump-sum way. When banks are bailed out, all of their liabilities are always repaid so none of their liabilities carry a risk premium. Hence the market does not perceive and price any risk. More precisely, the real price of deposits becomes $p = 1$ while the interbank market rate is

$$1 + i_m = \frac{1+\pi}{\beta} = (1+i).$$

The rest of the model is as before. In particular, and in addition to using (10), the first order conditions (11) and (12) become

$$k: \qquad F'(k)\left(1 - \frac{1}{2}q\right) = 1 + (i-r)\tau \tag{24}$$

$$q: \qquad (1-q)\,F(k) = k + (i-r)\tau k - (1+i)\lambda\tau k \tag{25}$$

An equilibrium with bail-out is a list (p, i_m, k, q, Q, m) such that given the bail-out policy, and policies i, τ, prices p and i_m, and aggregate risk $1-Q$, banks optimally choose m^b, firms choose k and q to maximize their surplus, $p = 1$, i_m clears the interbank market and $q = Q$. Combining (24) and (25) we obtain

$$q = 1 - \frac{\sigma\left[1 + (i-r)\tau - (1+i)\lambda\tau\right]}{(2-\sigma)\left(1 + (i-r)\tau\right) + \sigma(1+i)\lambda\tau} \tag{26}$$

$$F'(k) = (2-\sigma)\left(1 + (i-r)\tau\right) + \sigma(1+i)\lambda\tau \tag{27}$$

It should be clear that increasing τ and/or i will always decrease k and increase q.

$$Q = 1 - \frac{\sigma\left[1 + (i-r)\tau - (1+i)\lambda\tau\right]}{(2-\sigma)\left[1 + (i-r)\tau\right] + \sigma(1+i)\lambda\tau} \tag{28}$$

and investment k solves

$$F'(k) = \frac{\left[(2-\sigma)\left(1+(i-r)\tau\right)+\sigma\lambda(1+i)\tau\right]^2}{2(1-\sigma)\left[1+(i-r)\tau\right]+2\sigma\lambda(1+i)\tau} \tag{29}$$

Proposition 3. *In an equilibrium with bail-out, the investment level is k^b given by (27) and the quality of projects is $q = Q^b$ given by (26). k^b always declines with inflation or reserve requirements. Q^b increases with i and τ. For any $i \geq r$ and τ an anticipated bail-out policy is welfare improving.*

To complete the proof of Proposition 3, we now compare welfare under bail-out and no bail-out, say when $r = 0$. First, the quality choice is not affected by the bail-out. This is intuitive, because the equilibrium quality choice is not a function of market perception, and this is what the bail-out policy is affecting. Then compare the levels of investment as given by (17) and (27) with $r = 0$ and $\tilde{\tau} = \tau$. Notice that

$$F'(k) = F'(k^b)\frac{(2-\sigma)\left(1+(i-r)\tau\right)+\sigma\lambda(1+i)\tau}{2(1-\sigma)\left[1+(i-r)\tau\right]+2\sigma\lambda(1+i)\tau}$$

so that $k^b > k$ iff the fraction is greater than 1, or

$$1+(i-r)\tau \geq \lambda(1+i)\tau$$

which is the condition for an equilibrium to exist. Therefore, an (anticipated) bail-out policy is welfare improving because it does not affect risk-taking but it increases investment. Of course, this result hinges on the fact that a bail-out does not involve any distortion (such as the use of distortionary taxes).

4.3 Deposit insurance

In this section, we analyze whether an insurance scheme for deposits can do better than reserve requirements, or bail-out policies of the previous section. We now assume that banks have to work in the CM when they are born and pledge resources to the deposit insurance fund, as a fraction δ of their deposits k. In this sense, the insurance scheme is like a lending tax. However, it is more than a tax, as the deposit insurance would then tap into the funds to guarantee deposits (but not the interbank market loans). Sustainability of the deposit insurance mechanism requires that it has enough resources to cover the shortfall, i.e. $\delta k = (1-Q)pk(1-\tau(1+r))$ – where we still assume that all banks will hold τpk in reserves. Since in an equilibrium with deposit insurance deposits are as safe as money, $p = 1$ and we obtain $\delta = (1-Q)\,(1-\tau(1+r))$. Notice that banks always loose their contributions to the deposit insurance fund.

With such an insurance scheme in place, the bank's incentive constraint becomes

$$-\delta k + q\left[\phi + \tau(1+r)k - (1+i_m)(\tau k - m)\right] \geq Q(1+i_m)m$$

so that

$$\phi = \frac{\delta k + Q(1+i_m)m}{q} - (1+i_m)m + (i_m - r)\tau k$$

Then the bargaining problem becomes

$$\max_{k,\phi,q} qF(k) - \delta k - Q(1+i_m)m + q(1+i_m)m - q(i_m-r)\tau k - qk - \frac{1}{2}q^2 F(k),$$

with first order conditions

$$\begin{aligned} q\left(1 - \frac{1}{2}q\right)F'(k) &= \delta + q\left(i_m - r\right)\tau + q \\ (1-q)F(k) &= \left(i_m - r\right)\tau k + k - (1+i_m)m \end{aligned} \tag{30}$$

An equilibrium with deposit insurance is a list (p, i_m, k, q, Q, m) such that given the deposit insurance policy δ, and policies π, τ, prices p and i_m, and aggregate risk $1-Q$, banks optimally choose m, the contract k and q maximize the bank/firm's surplus, $p = 1$, i_m clears the interbank market and $q = Q$.

Market clearing condition still requires $\lambda\tau k = m$ and from the banks' demand for money,

$$1 + i_m = \frac{1+\pi}{Q\beta} = \frac{1+i}{Q}.$$

To solve for the equilibrium risk-return trade-off Q, we use $q = Q$, $\delta = (1-Q)\left(1-\tau(1+r)\right)$ and the expressions for m and i_m in the first order conditions to obtain

$$\left(1 - \frac{1}{2}Q\right) F'(k) = \frac{(1-Q)}{Q}(1-\tau(1+r)) + \left(\frac{1+i}{Q} - (1+r)\right)\tau + 1$$

$$(1-Q)F(k) = \left(\frac{1+i}{Q} - (1+r)\right)\tau k + k - \frac{1+i}{Q}\lambda\tau k$$

The fact that interbank market exposures are not covered by the deposit insurance scheme is reflected by the risk premium $\frac{1+i}{Q}$, while the contribution to the deposit insurance – increasing in the aggregate risk – is captured by the term $1/Q$. We can now find k and Q from re-arranging the first order conditions,

$$Q\left(1 - \frac{1}{2}Q\right) F'(k) = 1 + \tau(i - r) \tag{31}$$

where

$$Q = \frac{2\left\{(1-\sigma)\left[1+(i-r)\tau\right] + \sigma(1-Q)\left(1-(1+r)\tau\right) + \sigma\left(1+i\right)\lambda\tau\right\}}{(2-\sigma)\left[1+(i-r)\tau\right] + \sigma(1-Q)\left(1-(1+r)\tau\right) + \sigma\left(1+i\right)\lambda\tau} \tag{32}$$

There are two solutions to (32). We can show that the unique equilibrium is one with the negative root, as the other root is always greater than unity.

We can now compare the project's quality without insurance (16) with the one with insurance (32). For convenience, we restate (16) below

$$Q = \frac{2\left\{(1-\sigma)\left[1+(i-r)\tau\right] + \sigma(1+i)\lambda\tau\right\}}{(2-\sigma)\left[1+(i-r)\tau\right] + \sigma(1+i)\lambda\tau}$$

Notice that the RHS of (32) is strictly decreasing in Q whenever $1 > (1+r)\tau$. Also when $Q = 1$, the RHS of (16) equals the RHS of (32). Therefore, the RHS of (32) is always strictly higher than the RHS of (16) when $(1+r)\tau < 1$. Therefore, the RHS of (16) crosses the 45^o-

line at a lower point than the RHS of (32). This implies that, for given $\tau < 1/(1+r)$ and i, the equilibrium quality of projects is higher when there is deposit insurance. Finally, comparing the equilibrium level of investment without deposit insurance (18) with find that k is higher with deposit insurance.[14] We summarize this discussion in the following proposition.

Proposition 4. *Suppose $\tau(1+r) < 1$. There is a unique equilibrium with deposit insurance, the investment level is k given by (31) and the quality of projects is Q given by (32). The average quality, the level of investment, and welfare are higher with deposit insurance than without insurance.*

4.4 Capital requirements

In this section we show under which assumptions capital requirements are equivalent to liquidity requirements. As will become clear, capital and liquidity requirements are equivalent whenever raising equity is as "easy" as getting an interbank loans.

To show the equivalence result, we replace the liquidity requirement constraint by an equity constraint. Precisely we assume that for any loan size k, the bank has to have at least a fraction $\varepsilon \in [0, 1]$ of its investment in own equity. Therefore, the bank can finance a fraction $(1 - \varepsilon)$ of its loan with deposits and the remaining fraction with equity. Equity takes the form of cash accumulated by the bank when young. We relax the assumption that banks can produce sweat equity in the centralized market, and we instead assume that young banks raise equity by selling shares to suppliers. A share is a claim to the bank's future profit. Also, banks can sell additional shares to other banks once they meet a productive firm. This second equity market replaces the interbank market in the case with reserve requirements.

When there is inflation, it is costly to hold (unused) capital and so the equity requirement will bind. Then when a bank investing k raises ek equity from suppliers and Ek equity to other banks, it must be that $e + E = \varepsilon$. The bank's participation constraint given contract

[14] As Q is higher with deposit insurance, $Q(1 - Q/2)$ is also higher.

(k, ϕ), is

$$q\left[p(1-\varepsilon)k + \varepsilon k + \phi - (1+\varrho)Ek - p(1-\varepsilon)k\right] \quad \geq \quad Q(1+\rho)ek \tag{33}$$

The left hand side shows the expected profit of the bank that accrues to shareholders. If the firm fails, the bank's equity is wiped out. If the firm succeeds, it pays the principal $p(1-\varepsilon)k+\varepsilon k$ back to the bank plus ϕ, and the bank redeems deposits with a cost $-p(1-\varepsilon)k$. The bank also pays ϱ to other banks holding its equity. The right hand side is the outside option of the bank: It gets an expected return $Q(1+\rho)$ on its (existing) equity by buying other productive banks equity. Then we can simplify the participation constraint as

$$\phi + \varepsilon k \geq \left[\frac{Q(1+\rho)e}{q\varepsilon} + (1+\varrho)\frac{E}{\varepsilon}\right]\varepsilon k \tag{34}$$

The left hand side is the bank's resources when the firm succeeds, and the right hand side is the return paid to equity holders. In a symmetric equilibrium, $\varrho = \rho$ and $q = Q$.

We now consider the choice of contract (k, ϕ, q) . Again, it maximizes the firm's payoff,

$$\max_{k,\phi,q} q[F(k) - \phi - p(1-\varepsilon)k - \varepsilon k] - \frac{1}{2}q^2 F(k),$$

subject to the bank's participation constraint (34). Using the expression for ϕ, this problem becomes

$$\max_{k,q} qF(k) - \frac{1}{2}q^2 F(k) - q[(1+\varrho)Ek + p(1-\varepsilon)k] - Q(1+\rho)ek.$$

In the capital market, suppliers no longer expect banks to hold reserves, so that the price of deposits fully reflects the risk of bank's failure,

$$p = \frac{1}{Q} \tag{35}$$

This is one difference with reserves requirement: They help reduce the risk premium of deposits. Capital requirement do not. Then, the first order conditions of the firm's problem are

$$q\left(1-\frac{1}{2}q\right)F'(k) = q(1+\varrho)E + Q(1+\rho)e + \frac{q}{Q}(1-\varepsilon)$$
$$(1-q)\,F(k) = (1+\varrho)Ek + \frac{1}{Q}(1-\varepsilon)k$$

Therefore, we can already conclude that full equity requirement $\varepsilon = 1$ achieves the first best allocation whenever $\rho = 0$ and $E = 0$. The reason is that cash is cash: suppliers do not require a risk premium when they are paid with cash and there is no distortion of the allocation coming from this margin. Also, when $\rho = 0$ it is costless to build equity. Therefore, there is no distortion out of this margin either. Finally, when $E = 0$ the firm itself is not being held-up by the bank having to raise new equity, and it chooses the first best quality level.

We now solve for ρ. Since ρ is real the return on equity, we normalize the price of a bank share in the centralized market to one. When they consider how many shares s of a generic bank to purchase, suppliers solve the following problem in the centralized market,

$$\max_s -(1+\pi)s + \beta Q\left[\frac{p(1-\varepsilon)k + \varepsilon k + \phi - (1+\varrho)Ek - p(1-\varepsilon)k}{ek}\right]s$$

where Q is the supplier's belief about investment risk, and the term in square bracket is the real return on each share sold to suppliers, given the bank raises ek of equity from suppliers. In equilibrium $q = Q$, and (33) implies

$$1 + i = Q(1+\rho)$$

So suppose there is inflation, it must be that banks are compensated for building equity, so that $1 + i = 1 + \rho$. In case $i > \rho$, the holdup problem implies that there is no equilibrium, as the bank has no incentive to build equity ahead of its lending activity.[15] Also, the higher i the more expensive it is to raise equity (the higher the return on equity has to be, so the lower investment is. Finally, whenever $i > 0$ banks have to raise equity from other banks, as

[15]This would change if banks had some bargaining power.

otherwise there would not be any equilibrium (because of the hold up problem). Hence, it must be that $e < \varepsilon$ (unless $i = 0$.) The equilibrium condition for the (inter)bank market for equity gives $(1-\lambda)e = \lambda E$. Together with $e + E = \varepsilon$, we obtain

$$e = \lambda\varepsilon$$

Therefore, in equilibrium

$$\begin{aligned} Q\left(1-\frac{1}{2}Q\right)F'(k) &= 1 + i\varepsilon \\ Q\left(1-Q\right)\frac{F(k)}{k} &= 1 + i\varepsilon - (1+i)\lambda\varepsilon \end{aligned}$$

Comparing these two equations with (18) and (19), it is straightforward to see that setting $\varepsilon = \tau$ gives us the same equilibrium condition as in the case with reserve requirements. So, in some sort of Modigliani-Miller way, liquidity requirements are equivalent to capital requirement.

5 Literature review

Williamson (1999) argues that the creation of tradable deposit allows productive intermediation and is thus desirable. Using a similar angle of attack, Chari and Phelan (2016) argue that the creation of deposits has the (private) benefits of insuring against liquidity shocks, while at the same time imposing a pecuniary externality by raising the price level. This implies that the social benefits of deposit creation can even be negative. As a result 100% reserve requirement can be desirable. Our mechanism also plays through a pecuniary externality, but while Chari and Phelan study the effect of consumption loans, we study the effect of corporate credit lines on the production process. Then we can show that deposits possibly increase leverage beyond its optimal level and increasing risk (in addition to the price level). Monnet and Sanches (2015) also show that 100% reserve requirements may be undesirable because bankers cannot commit to repay deposits. Instead, our results are driven by limited liability. Still with limited commitment but with moral hazard for banks decision, Hu and Li (2017) analyze the effect of capital regulation. Instead, we concentrate on the effects of monetary policy on banks' balance sheet risk. Sanches (2015) argues that a purely private monetary regime is inconsistent with macroeconomic stability. The result hinges on endogenously determined limits on private money creation and the presence of self-fulfilling equilibrium characterized by monetary collapse.

Jakab and Kumhof (2015) remark that, with a few exceptions, the academic literature has focused on a debatable model of banks, namely the "intermediation of loanable funds" model. In this model banks intermediate funds from savers to borrowers. A prime example of such a model is Diamond and Dybvig (1983), or Berentsen, Camera, and Waller (2007). Calomiris, Heider, and Hoerova (2015) is also using the "intermediation" model but is more related to our question, as they analyze the need for liquidity requirements for banks. In their model, liquidity requirements act as a disciplining device for bankers who otherwise would engage in moral hazard. Instead, we analyze the effect of liquidity requirements on investment and we provide a general equilibrium model where banks can create deposits that circulate as means of payments. Then we can analyze the effect of monetary policy in bank risk taking. Jakab and Kumhof argue that banks' main activity is to finance firms through the creation of money (or deposits). Among many other results, they show that the "financing" model of banking explains why leverage is pro-cyclical. Our model belongs to the financing view of banking and we concentrate on risk taking and the optimal reserve requirement policy and its interaction with monetary policy when banks issue tradable deposits.[16] Our paper is also related to Williamson (2016) that features the moral hazard problem of creating low quality collateral when the interest rate is low.

6 Conclusion

We presented a model to study the implications of deposit making on risk taking. We believe our arguably simple model captures several important features of bank lending activities: Firms need funding and they obtain it from banks. Banks finance firms by creating deposits.

[16] We refer the reader to Bigio and Weill (2016) for a recent theory of banks balance sheet and why banks are useful in providing liquid assets.

Deposits are used as means of payments. Deposits carry a risk premium as long as they are not insured or only partially backed by liquid assets. We find that borrower's quality is an increasing function of inflation: The increased cost of liquid asset induces banks to charge higher rates to borrowers. As a consequence, they borrow less and their debt level falls. As a result, they take less risk.

The model is simple and we chose to abstract from many relevant aspects. Let us mention the four most obvious: First, banks have no bargaining power. While this seems unrealistic, this assumption implies that the firms' incentives are most aligned with the one of a planner. If banks had some bargaining power, it would only deter firms from choosing higher quality as banks would capture some of the surplus. Second, banks do not take deposits from depositors and they finance their liquidity requirements using only (sweat) equity. Hence, we cannot study issues such as bank runs in the current version of the model. Still let us stress that the risk premium on deposits is getting to the idea of a run on banks: If the risk premium increases to infinity, firms cannot trade deposits. We think it would be interesting to extend the model in this direction. The third aspect that is missing from the model is the cost of raising equity. Here, we modeled equity as an effort level that banks have to exert in order to "get started". Modeling bank equity as shares would also be an interesting extensions. Finally, analyzing growth should yield interesting insights in particular regarding the debate on growth versus stability of the financial system. Overall, we expect the mechanism we highlighted to be robust to these four, and other extensions.

7 Bibliography

Berentsen, A., G. Camera, C. Waller, (2007) "Money, Credit, and Banking", *Journal of Economic theory* 135 (1), 171-195.

Chari, V. V., and C. Phelan (2014) "On the social usefulness of fractional reserve banking," *Journal of Monetary Economics*, 65, 1-13.

Calomiris, W. C., F. Heider, and M. Hoerova (2015) "A theory of bank liquidity requirements," Working paper, ECB.

Jakab, Z., and M. Kumhof (2015) "Banks are not intermediaries of loanable funds – and why this matters," Bank of England Working Paper No. 529. May 2015.

Hu, T.-W., and Y. Li (2017) "Optimal Banking Regulation with Endogenous Liquidity Provision," mimeo.

Monnet, C., and D. Sanches (2015) "Private money and banking regulation," *Journal of Money, Credit, and Banking* 47 (6), 1031-1062.

Rocheteau, G., R. Wright, and C. Zhang (2016) Corporate finance and monetary policy, mimeo, Purdue University.

Sanches, D. (2015) "On the inherent instability of private money," *Review of Economic Dynamics* 20, 198-214.

Williamson, S. (1999) "Private Money," Journal of Money, Credit, and Banking 31(3), 469-491.

Williamson, S. (2016) "Low Real Interest Rates, Collateral Misrepresentation, and Monetary Policy," Federal Reserve Bank of St Louis, Working Paper 2014-026B.

A Proof of Proposition 2

When $i \simeq r$, (21) implies that welfare is just a function of k. When $i \simeq r$, (17) gives k as

$$F'(k) = \frac{[(2-\sigma) + \sigma\lambda\tau]^2}{2(1-\sigma) + 2\sigma\lambda\tau}$$

which RHS is always decreasing in τ. By concavity of the production function k, and thus welfare, is strictly increasing with τ. Hence τ is set optimally to 1. For the second part of the proof, set $r = 0$. We want to show that as i becomes large, then $\frac{\partial \mathcal{W}}{\partial \tau}$ evaluated at $\tau = 1$ is negative. With $r = 0$ we have

$$\frac{\partial \mathcal{W}}{\partial \tau} = \lambda \left[\frac{1-\sigma+i\tau}{\sigma}\right] \frac{\partial k}{\partial \tau} + \frac{\lambda k i}{\sigma}.$$

Using (17) and after some algebra we obtain

$$\frac{dk}{d\tau} |_{\tau=1} \quad = \quad \frac{(2-(1-\lambda)\sigma)\left[i(1-(1-\lambda)\sigma)(2-(1-\lambda)\sigma) - (1-\lambda)\lambda\sigma^2\right]}{2(1-(1-\lambda)\sigma)^2 F''(k)}$$

When $i = 0$ then

$$\frac{dk}{d\tau} |_{\tau=1} = -\frac{(2-(1-\lambda)\sigma)\,(1-\lambda)\lambda\sigma^2}{2(1-(1-\lambda)\sigma)^2 F''(k)} > 0$$

while when $i = \infty$ then

$$sign\left(\frac{dk}{d\tau} |_{\tau=1}\right) \to sign\left(\frac{(2-(1-\lambda)\sigma)^2\,(1-(1-\lambda)\sigma)}{2(1-(1-\lambda)\sigma)^2 F''(k)}\right) < 0$$

So the first term in $\frac{\partial \mathcal{W}}{\partial \tau}$ is negative if i is large enough and $\tau = 1$. Now let us look at the term ki. Since k tends to zero when $i \to \infty$, we need to use L'Hospital's rule. Then

$$
\begin{aligned}
\lim_{i\to\infty} ki &= \frac{\lim_{i\to\infty} k'(i)}{\lim_{i\to\infty} \partial(1/i)/\partial i} = \frac{\lim_{i\to\infty} -\frac{2^{\frac{1}{1-\sigma}}\left(\frac{(1+i)(2-(1-\lambda)\sigma)^2}{\sigma-(1-\lambda)\sigma^2}\right)^{\frac{1}{-1+\sigma}}}{(1+i)(1-\sigma)}}{\lim_{i\to\infty} -(1/i^2)} \\
&= \lim_{i\to\infty} \frac{2^{\frac{1}{1-\sigma}}\left(\frac{(2-(1-\lambda)\sigma)^2}{\sigma-(1-\lambda)\sigma^2}\right)^{\frac{1}{-1+\sigma}}}{(1-\sigma)} \frac{i^2}{(1+i)^{\frac{2-\sigma}{1-\sigma}}} \approx i^{2-\frac{2-\sigma}{1-\sigma}} = i^{\frac{-\sigma}{1-\sigma}} = 0
\end{aligned}
$$

where the last equality follows from $0 < \sigma < 1$. Hence, ki tends to zero as $i \to \infty$. As a result, if i is large enough, we have

$$
\frac{\partial \mathcal{W}}{\partial \tau} |_{\tau=1} = \lambda \left[\underbrace{\frac{1-\sigma+i\tau}{\sigma}}_{>0} \right] \underbrace{\frac{\partial k}{\partial \tau}}_{<0} + \underbrace{\frac{\lambda k i}{\sigma}}_{\sim 0} < 0,
$$

i.e. $\tau < 1$ is optimal.

B Financing firms with deposits

In this Appendix, we show that banks only finance firms with deposits whenever $i > r$ and that they only demand central bank money in order to satisfy their reserve requirement. To do so, we need to consider four cases: (1) banks do not borrow on the interbank market. Second banks borrow on the interbank market, and (2) banks do not default on any of their liabilities, (3) banks do not default on their deposits but partially default on their junior interbank liabilities, and (3) banks default on their deposits (and so on their interbank liabilities).

A.1 Banks do not borrow on the interbank market

Then the outside option of banks is to earn interest on reserves:

$$\mathcal{O}(m^b) = (1+r)m^b$$

As the firm takes all, the bank expects to receive a payoff equal to $(1+r)m^b$. So, since $i > r$, the bank will not bring any cash to the banking market. With positive reserve requirements, the only equilibrium is autarky.

A.2 Banks borrow on the interbank market

Since the interbank market is active, the outside option of banks is to lend on the interbank market. Then we have to consider three cases.

A.2.1 No default

The no-default condition is

$$(1+r)\left(m^b + b - k^o\right) - (1+i_m)\, b \quad \geq \quad p^n k^n \tag{36}$$

. In this case, banks do not default on their deposits so that $p^n = p^0 = 1$. Therefore, banks' outside is to lend risk free on the interbank market, with a return

$$\mathcal{O}(m^b) = (1+i_m)\, m^b.$$

Thus, it follows that the bank's problem with respect to m^b is

$$\max_{m^b} \left\{-(1+\pi)m^b + \beta\,(1+i_m)\, m^b\right\}$$

$1+i_m = \frac{1+\pi}{\beta} = 1+i > 1+r$. Using $p^n = 1$, the bargaining problem is

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^o,b,\phi} \left\{ q \left[\left(1-\frac{1}{2}q\right) F(k^n+k^o) - \phi - k^n - k^o \right] \right\}$$

subject to

$$\tilde{\tau} k^n \leq (1+r)\left(m^b + b - k^o\right),$$

$$q\left[k^n + k^o + \phi\right] + (1+r)\left(m^b + b - k^o\right) - (1+i_m)\, b - k^n = (1+i_m) m^b,$$

and $k^o, k^n \geq 0$.

Given (36) and $p^n = 1$, the reserve constraint cannot bind as $\tilde{\tau} \leq 1$. However, then the FOC with respect to b gives $1+r = 1+i_m$, which contradicts $i > r$. Therefore there cannot be an equilibrium where banks do not default on their interbank loans and $i > r$.

A.2.2 No default on deposits, but default on interbank loans

Banks do not default on their deposits whenever

$$p^n k^n \leq (1+r)(m^b + b - k^o)$$

but they default on their interbank loans whenever

$$(1+r)(m^b + b - k^o) < p^n k^n + (1+i_m)\, b$$

Since banks do not default on their deposits, $p^n = 1$. Therefore banks' outside option is

$$\mathcal{O}(m^b) = Q\,(1+i_m)\, m^b + (1-Q)\left[(1+r)\left(\tilde{m}^b + \tilde{b} - \tilde{k}^o\right) - \tilde{k}^n\right] \frac{m^b}{\tilde{b}}$$

Plugging this back in the problem of the young bank in the centralized market, we obtain $i_m > i$ (this only holds with equality if banks do not default on their interbank loans.) Using

$p^n = 1$, the bargaining problem is given by

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^0,b^1,\phi} \left\{ q \left[\left(1 - \frac{1}{2}q\right) F(k^n + k^o) - \phi - k^n - k^o \right] \right\}$$

subject to

$$\tilde{\tau} k^n \leq (1 + r) \left(m^b + b - k^o\right),$$

$$q \left[k^n + k^0 + \phi + (1 + r) \left(m^b + b - k^o\right) - (1 + i_m) b - k^n\right] = \mathcal{O}(m^b),$$

and $k^0, k^n \geq 0$. Again, as the bank does not default on deposits, the reserve requirement constraint is not binding. Using the IC of the bank to replace for ϕ in the objective function, it becomes

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^0,b^1,\phi} q \left(1 - \frac{1}{2}q\right) F(k^n{+}k^o){-}\mathcal{O}(m^b){+}q \left[(1 + r) \left(m^b + b - k^o\right) - (1 + i_m) b - k^n\right]$$

and the FOC with respect to b gives $r = i_m$. However, this cannot be an equilibrium because $i_m > i > r$.

A.2.3 Default on deposits and interbank loans

Bans default on their deposits whenever

$$p^n k^n \quad \geq \quad (1 + r) \left(m^b + b^1 - k^0\right)$$

The outside option of banks is

$$\mathcal{O}(m^b) = Q \left(1 + i_m\right) m^b$$

So the banks problem yields $1 + i_m = (1 + i)/Q$. The bargaining problem is given by

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^o,b,\phi} \left\{ q \left[\left(1 - \frac{1}{2}q\right) F(k^n + k^o) - \phi - p^n k^n - k^o \right] \right\}$$

subject to

$$\tilde{\tau}k^n \leq (1+r)\left(m^b + b - k^0\right),$$

$$q\left[p^n k^n + k^o + \phi + (1+r)\left(m^b + b - k^o\right) - (1+i_m)\,b - p^n k^n\right] = (1+i)m^b,$$

and $k^o, k^n \geq 0$. Isolating ϕ in the last constraint and replacing in the objective function yields

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^o,b,\phi} q\left(1-\frac{1}{2}q\right)F(k^n+k^o) - (1+i)m^b + q\left[(1+r)\left(m^b+b-k^o\right) - (1+i_m)\,b - p^n k^n\right]$$

such that

$$\tilde{\tau}k^n \leq (1+r)\left(m^b + b - k^0\right)$$

Since $i > r$, it must be that the reserve requirement constraint is binding. Otherwise the first order condition with respect to b would give $i = r$ (see above). Therefore, we can use the constraint to replace b in the objective function. Then it becomes,

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^o,\phi} q\left(1-\frac{1}{2}q\right)F(k^n+k^o) - (1+i)m^b + q\left[\tilde{\tau}k^n - (1+i_m)\,\frac{\tilde{\tau}k^n - (1+r)(m^b - k^0)}{(1+r)} - p^n k^n\right]$$

The first order conditions are

$$\begin{aligned}
\boldsymbol{q} &: \ (1-q)\,F(k^n+k^o) + \tilde{\tau}k^n - (1+i_m)\,\frac{\tilde{\tau}k^n - (1+r)(m^b-k^0)}{(1+r)} - p^n k^n = 0 \\
\boldsymbol{k^n} &: \ q\left(1-\frac{1}{2}q\right)F'(k^n+k^o) - q\left[\frac{(i_m - r)}{(1+r)}\tilde{\tau} + p^n\right] + \lambda_{k^n} = 0 \\
\boldsymbol{k^0} &: \ q\left(1-\frac{1}{2}q\right)F'(k^n+k^o) - q\,(1+i_m) + \lambda_{k^o} = 0
\end{aligned}$$

Since the reserve requirement binds, suppliers charge a risk premium, and

$$p^n = \frac{1}{Q} - \frac{1-Q}{Q}\,(1+r)\,\frac{m^b + b^1 - k^0}{k^n} = \frac{1}{Q} - \frac{1-Q}{Q}\tilde{\tau}$$

and the FOC with respect to k^n becomes

$$\boldsymbol{k^n}: \qquad q\left(1-\frac{1}{2}q\right)F'(k^n+k^o)-\frac{q}{Q}\left[\frac{(1+i)}{(1+r)}\tilde{\tau}+1-\tilde{\tau}\right]+\lambda_{k^n} \quad = \quad 0$$

while the FOC with respect to k^0 becomes

$$\boldsymbol{k^0}: \qquad q\left(1-\frac{1}{2}q\right)F'(k^n+k^o)-\frac{q}{Q}(1+i)+\lambda_{k^o} \quad = 0$$

Therefore $\lambda_{k^0} > \lambda_{k^n} = 0$ whenever

$$(1+i) > \frac{(1+i)}{(1+r)}\tilde{\tau}+1-\tilde{\tau}$$

The RHS is a weighted average of 1 and a number less than or equal to $1+i$. Therefore the RHS is always less than $1+i$. So we conclude that whenever $i > r \geq 0$ the $k^0 = 0$ and $k^n > 0$.